The Study on Block Compound Matrix Inclusion Region of Inhomogeneous Block Eigenvalue under Bipartition

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Abstract: Matrix Eigenvalues Have Important Application Value in Modern Scientific Research and Engineering Calculation. Studying the Matrix Eigenvalue Calculation Method and Giving Its Corresponding Inclusion Domain Helps Matrix Analysis. Based on This, This Paper Briefly Summarizes the Distribution of Eigenvalues of the Block Composite Matrix, the Eigenvalue Inclusion Domain and So on. in the Research Process, the Concept of Block Composite Matrix and Block Composite Matrix Eigenvalues is Introduced. the Domain Eigenvalues of the Block Eigenvalues of the Bipartition Complex Matrix Are Studied to Provide a Reference for the Study of the Bipartition Composite Matrix.

1. Introduction

1.1 Literature Review

Li Yaotang and Chen Gang clearly defined the block double α1-matrix and the block double α2-matrix, and considered that compared with the Gerschgorin and Brauer eigenvalue inclusion regions, the eigenvalue inclusion regions of the double α1-matrix and the double α2-matrix can be more Accurately determine the location of matrix eigenvalues (Li and Chen, 2013). In the research, Xu Changling used the block composite matrix and block eigenvalue definition, and used the common eigenvalue inclusion domain theory in the calculation process to study the block eigenvalue inclusion domain of the block composite matrix. At the same time, by combining the local block double diagonal dominant definition and the application containing domain theory, Xu Changling clearly defines the domain range of the block eigenvalues (Xu, 2015). Matrix eigenvalues play a key role in the calculation and research process. Traditional algorithms generally use circular or elliptical regions to represent the range of regions. Cao Haisong and Wu Junliang sought a new method to represent the domain range of the complex matrix eigenvalues, and proposed that any eigenvalues of the matrix of the nth order with real coefficient feature polynomials are included in $\alpha_2(x-trA/n)2+\beta_2 y_2 \le \alpha_2\beta_2$,and $\alpha_2(x-trA/n)2+\beta_2 y_2 \le \alpha_2\beta_2$ is the given ellipse region (Cao and Wu, 2012). H-matrix is a kind of matrix which is more common and very special in calculation. It plays a very important role in the field of linear algebra calculation. And H-matrices also play an important role in the study of block composite matrices. Cui Lina made a brief statement on the concept of nonsingular H-matrices, and gave sufficient conditions for the calculation of nonsingular H-matrices. It is proposed that this condition should be widely promoted and continuously improved in algebra (Cui, 2013). . Li Yanyan studied the new bounds of eigenvalues of M-matrices. By setting the eigenvalues of the matrix eigenvalues to include the upper bounds of the non-primitive diagonal elements of A-1, the inverse matrix A of strictly diagonally dominant M-matrices A is proposed. New Territories of -1 element (Li, 2014).

1.2 Purpose of the Study

With the continuous development of mathematical theory, the research of composite matrix has developed rapidly, and its position in the fields of engineering technology and natural science is rising. In the study of block composite matrix theory, data operation is a key link. At this stage, matrix-related problems often occur in actual production work and engineering technology

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calculations, and these problems can be solved by the composite matrix. Therefore, studying composite matrices is of great significance for solving real problems. In this paper, the theoretical basis of block composite matrix and block eigenvalue inclusion domain is discussed in detail. The eigenvalue inclusion domain of block composite matrix is preliminarily conditioned, in order to provide the eigenvalue inclusion domain study of block composite matrix block under two parts. The new theoretical basis promotes the in-depth development of composite matrix research.

2. Theoretical Basis

In the field of linear algebra research, eigenvalues are one of the core concepts. In the fields of computer, mathematics, physics, etc., eigenvalues are widely used. The basic definition is as follows: Suppose A is an n-order matrix. If there is a constant k and an n-dimensional column vector X, and AX=kX can be made, then k can be called the eigenvalue of matrix A (Zhang and Shang, 2018).

The so-called matrix refers to a set of real or complex numbers arranged according to the array, which is the earliest from the matrix of equations and constants (Cao et al, 2013). The emergence of matrices has a great impact on scientific computing, and its technology specifically includes addition, subtraction, multiplication, division, transposition, and conjugate. According to specific characteristics, the matrix can be divided into various types, such as a symmetric matrix, an orthogonal matrix, a composite matrix, a triangular matrix, and the like. Among them, the composite matrix has a strong application space in the qualitative theory of differential equations.

In this paper, we use the idea of blocking to focus on the eigenvalue inclusion domain of the block-composite matrix block of the two parts. $A_n(c)$ represents all n-order composite squares that can be exchanged in pairs.. $M_{p,q}(p_n)$ is divided into $p*q$ block, each sub-block belongs to the $P_n(c)$ matrix set.

3. Preparatory Lemma

In the proof process, the following two lemmas are used.

Lemma 1: $B=(B_{ij})\in M_m(p_n)$, α, β are $<m>$ bipartitions, $|\alpha|=p,|\beta|=m-p.$ .If B_{ii}, $\mathrm{i}\in\langle m\rangle$ is not singular, B is singular and satisfied :

$$\left\|B_{\mathrm{ii}}^{-1}\right\| > \max\{R_{(i)}^{\alpha}(B)\}\forall i\in\alpha, \tag{1}$$

$$\left\|B_{\mathrm{jj}}^{-1}\right\| > \max\{R_{(j)}^{\beta}(B)\}\forall j\in\beta, \tag{2}$$

Then there exists $\mathrm{X}=(\mathrm{X}_1,\mathrm{X}_2,...,X_m)^T\in C^{mn}\setminus\{0\}$, $X_i\in C^n$ so that $B\mathrm{X}=0$.

$$\text{Let } \left\|\mathrm{X_K}\right\|=\max_{i\in\alpha}\{\left\|X_i\right\|\},\left\|\mathrm{X_l}\right\|=\max_{j\in\beta}\{\left\|X_j\right\|\},\max_{i\in\prec m>}\{\left\|X_i\right\|\}=1,$$

then $y=y(y_{j1j2})\in Q_{p(m-p)}=\{(\alpha_{1,...}\alpha_{p(m-p)})^T\in R^{p(m-p)},a_i\in[0,1],i\in<p(m-p)>\}$,

In the formula, $(j1,j2)\in\Omega=\{(i,j):i\in\alpha,j\in\beta\}$,

$$\left\|\mathrm{A}_{kk}^{-1}\right\|^{-1}-R_K^{\alpha}(B))(\left\|B_{LL}^{-1}\right\|^{-1}-R_L^{\beta}(B))\leq\sum_{(j1,j2)\in\Omega}\left\|B_{\mathrm{kj2}}\right\|\left\|B_{lj1}\right\|y_{j1j2}\,.$$

Proof: $B\mathrm{X}=0$ will be expanded according to the Kth line:

$$B_{K1}X_1+B_{K2}X_2+...+B_{KM}X_M=0,$$

then $$B_{\mathrm{KK}}\mathrm{X_K}=-\sum_{\mathrm{j1}\in\alpha\{\mathrm{K}\}}B_{\mathrm{KJ1}}X_{J1}-\sum_{\mathrm{j2}\in\beta\{\mathrm{K}\}}B_{\mathrm{KJ2}}X_{J2}.$$

B_{KK} is not strange, there is $X_K = -\sum_{J1\in\alpha\setminus\{K\}} B_{KK}^{1} B_{KJ1} X_{J1} - \sum_{J2\in\alpha\setminus\{K\}} B_{KK}^{-1} B_{KJ2} X_{J2}$.

Further known:

$$\|X_K\|\|B_{KK}^{-1}\|^{-1} \le \sum_{J1\in\alpha\setminus\{K\}} \|B_{KJ1}\|\|X_K\| + \sum_{J2\in\beta} \|B_{KJ2}\|\|X_L\|$$

$$\le R_K^{(\alpha)}(B)\|X_K\| + R_K^{(\beta)}(B)\|X_L\|, \tag{3}$$

Similarly, $BX = 0$ expands by line L:

$$\|X_L\|\|A_{LL}^{-1}\|^{-1} \le \sum_{J1\in\alpha\setminus\{K\}} \|B_{LJ1}\|\|X_{J1}\| + \sum_{J2\in\beta\setminus\{L\}} \|B_{KJ2}\|\|X_{J2}\|$$

$$\le R_L^{(\alpha)}(B)\|X_L\| + R_L^{(\beta)}(B)\|X_L\| \tag{4}$$

then $\|X_K\|\rangle 0, \|X_L\|\rangle 0$,Otherwise if $\|X_L\| = 0$, $\|X_K\| = \max_{i\in<m>}\|X_i\| = 1$.

Known by (3), $B_{KK}^{-1} \le R_K^{(\alpha)}(B)$, $K\in\alpha$,this contradicts the known (1).

Similarly, $\|X_K\| \ge 0$,Assume $1 = \|X_k\| \ge \|X_l\| > 0$.

Known by (1), (2), (3) and (4):

$$\left(\|B_{kk}^{-1}\|^{-1} - R_k^{(\alpha)}(B)\right)\left(\|B_{ll}^{-1}\|^{-1} - R_l^{(\beta)}(B)\right)$$

$$\le \left(\sum_{j_2\in\beta}\|B_{kj_2}\|\frac{\|X_{j_2}\|}{\|X_k\|}\right)\left(\sum_{j_1\in\alpha}\|B_{lj_1}\|\frac{\|X_{j_1}\|}{\|X_l\|}\right)$$

$$\le \sum_{(j_1,j_2)\in\Omega}\|B_{kj_2}\|\|B_{lj_1}\|\frac{\|X_{j_2}\|}{\|X_k\|}\frac{\|X_{j_1}\|}{\|X_l\|}.$$

make

$$y_{j_1j_2} = \frac{\|X_{j_1}\|\|X_{j_2}\|}{\|X_k\|\|X_l\|},$$

Then there exist $y_{j_1j_2} \in [0,1], \forall (j_1, j_2) \in \Omega$,and

$$\left(\|A_{kk}^{-1}\|^{-1} - R_k^{(\alpha)}(B)\right)\left(\|B_{ll}^{-1}\|^{-1} - R_t^{(\beta)}(B)\right) \le \sum_{(j_1,j_2)\in\Omega}\|B_{kj_2}\|\|B_{lj_1}\|y_{j_1j_2}$$

Lemma 2: $B = (B_{ij}) \in M_m(P_n)$, α , β are $<m>$ bipartitions, $|\alpha| = p, |\beta| = m-p$,

then there exists $y = (y_{j_1j_2}) \in Q_{p(m-p)}, (j_1, j_2) \in \Omega$

so:

$$\sum_{(j_1j_2)\in\Omega}\left[\left(\|B_{j_1j_1}^{-1}\|^{-1} - C_{j_1}^{(\alpha)}\right)\left(\|B_{j_2j_2}^{-1}\|^{-1} - C_{j_2}^{(\beta)}\right) - C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}\right]y_{j_1j_2} \le C_k^{(\beta)}C_l^{(\alpha)} - \left(\|B_{kk}^{-1}\|^{-1} - C_k^{(\alpha)}\right)\left(\|B_{ll}^{-1}\|^{-1} - C_l^{(\beta)}\right)$$

prove: Because $BX = 0$, $\forall i\in\alpha$

$$B_{ii}X_i = -\sum_{j\ne i} B_{ij}X_j = -\sum_{j_1\in\alpha\setminus\{i\}} B_{ij_1}X_{j_1} - \sum_{j_2\in\beta} B_{ij_2}X_{j_2}$$

Because $B_{ii}(i\in\langle m\rangle)$ is not singular, so $\|X_i\|\|B_{ii}^{-1}\|^{-1} - \sum_{j_1\in\alpha\setminus\{i\}}\|B_{ij_1}\|\|X_{j_1}\| \le \sum_{j_2\in\beta}\|B_{ij_2}\|\|X_{j_2}\|$,

Further available:

$$\sum_{i\in\alpha}\|X_i\|\|B_{ii}^{-1}\|^{-1}-\sum_{i\in\alpha}\sum_{j_1\in\alpha\setminus\{i\}}\|B_{ij_1}\|\|X_{j_1}\|\le\sum_{i\in\alpha}\sum_{j_2\in\beta}\|B_{ij_2}\|\|X_{j_2}\|,$$

Which is :

$$\sum_{i\in\alpha}\|X_i\|\|B_{ii}^{-1}\|^{-1}-\sum_{j_1\in\alpha}C_{j_1}^{(\alpha)}(B)\|X_{j_1}\|\le\sum_{j_2\in\beta}C_{j_2}^{(\alpha)}(B)\|X_{j2}\|,$$

$$\sum_{j_1\in\alpha}\left(\|B_{j_1j_1}^{-1}\|^{-1}-C_{j_1}^{(\alpha)}(B)\right)\|X_{j_1}\|\le\sum_{j_2\in\beta}C_{j_2}^{(\alpha)}(B)\|X_{j_2}\| \tag{5}$$

Similarly:

$$\sum_{j2}\left(\|B_{j2j2}^{-1}\|-C_{j2}^{(\beta)}(B)\right)\|X_{j2}\|\le\sum_{j1}C_{j1}^{(\beta)}(B)\|X_{j1}\| \tag{6}$$

From (5), (6) there are:

$$\sum_{(j2,j2)\in\Omega\setminus\{k,l\}}\left[\left(\|B_{j1j1}^{-1}\|^{-1}-C_{j1}^{(\alpha)}\right)\left(\|B_{j2j2}^{-1}\|^{-1}-C_{j2}^{(\beta)}\right)-C_{j1}^{(\beta)}C_{j2}^{(\alpha)}\right]\frac{\|X_{j1}\|\|X_{j2}\|}{\|X_k\|\|X_l\|}$$
$$\le C_l^{(\alpha)}C_k^{(\beta)}-\left(\|B_{kk}^{-1}\|^{-1}-C_k^{(\alpha)}\right)\left(\|B_{ll}^{-1}\|^{-1}-C_l^{(\beta)}\right)$$

Make $y_{j_1j_2}=\frac{\|X_{j1}\|\|X_{j2}\|}{\|X_k\|\|X_l\|}$, $(j_1,j_2)\in\setminus\Omega\{(k,l)\}$, $y_{kj}=0$,Then there exists $y_{j_1j_2}\in[0,1]$, $\forall(j_1,j_2)\in\Omega$,and the conditions of the theorem are satisfied, and the proof is completed.

4. The Study on Block Compound Matrix Inclusion Region of Inhomogeneous Block Eigenvalue under Bipartition

Theorem 1: $B=(B_{ij})\in M_m\ (P_n)$, α and β are $<m>$ bipartitions, $|\alpha|=p,|\beta|=m-p$,

If it is arbitrary (k,l)□ Ω, $(\overline{S_1^-}\cap\overline{S_2^+})\cap Q_{p(m-p)}=\Phi$,Then B is Non-singular.

Theorem 2: $B=(B_{ij})\in M_m\ (P_n)$, α and β are $<m>$ bipartitions,

$\alpha\cup\beta=<m>,\alpha\cap\beta=\Phi$

If $J_\alpha(B)=\{i\in\alpha:\|B_{ii}^{-1}\|^{-1}>R_i^{(\alpha)}\}\cup J_\beta(B)=\{j\in\beta:\|B_{jj}^{-1}\|^{-1}>R_j^{\beta}\}\ne\Phi$

And ∀（i，j）$\in\Omega$,

$$(\|B_{ii}^{-1}\|^{-1}-R_i^{\alpha})\left(\|B_{jj}^{-1}\|^{-1}-R_j^{(\beta)}\right)>R_i^{(\beta)}R_j^{(\alpha)}, \tag{7}$$

Then B is Non-singular.

Proof: by (7) $J_\alpha(B)\ne\Phi$,and $J_\beta(B)\ne\Phi$ .Otherwise, if $J_\alpha(B)=\Phi$,then $\forall i\in\alpha$,and $|B_{ii}^{-1}\|^{-1}\le R_i^{(\alpha)}$.

From (7) the right side is greater than or equal to zero, $\forall j\in\beta$ Must have $|B_{jj}^{-1}\|^{-1}\le R_j^{(\beta)}$,which is $J_\beta(B)=\Phi$,contradictory with known. J_α corresponds to α, J_β corresponds to β,so J_∂ and J_β are $<m>$ bipartitions.

Also because $J_\alpha(B)\cup J_\beta(B)\ne\Phi$

This means $J_\alpha(B)=\alpha$, $J_\beta(B)=\beta$ .And then ∀（i，j）$\in\Omega$, there is

$(\|B_{ii}^{-1}\|^{-1}-R_i^{\alpha})\left(\|B_{jj}^{-1}\|^{-1}-R_j^{(\beta)}\right) > \sum_{(j_1,j_2)\in\Omega}\|B_{ij2}\|\|B_{jj1}\| \geq \sum_{(j_1,j_2)\in\Omega}\|B_{ij2}\|\|B_{jj1}\| y_{j1j2}$, $\forall y_{j1j2}\in[0,1]$,then $\overline{S_1^-}\cap Q_{p(m-p)}=\Phi$ .It can be seen from Lemma 1 that it is Non-singular.

Theorem 3: $B=(B_{ij})\in M_m(P_n)$, α, β are $<m>$ bipartitions, If there is $t\in<m>$ make: $(\|B_{ii}^{-1}\|^{-1}-R_i^{\alpha})\left(\|B_{jj}^{-1}\|^{-1}-R_j^{(\beta)}\right) > R_i^{(\beta)}R_j^{(\alpha)}$, $\forall$ (i, j) $\in\Omega$, (8)

$$\sum_{(j_1,j_2)\in T_t}[(\|B_{j_1j_1}^{-1}\|^{-1}-C_{j_1}^{(\alpha)})(\|B_{j_2j_2}^{-1}\|^{-1}-C_{j_2}^{(\beta)}) > \sum_{(j_1,j_2)\in T_t}C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}, \forall T_t\subseteq\Omega, \tag{9}$$

Then B is Non-singular.

Proof: Define the hyperplane:

$$P=\left\{y=(y_{j_1j_2})\in Q_{p(m-p)}, (j_1,j_2)\in\Omega: \sum_{(j_1j_2)\in\Omega}y_{j_1j_2}=t-1\right\},$$

Then p divides $Q_{p(m-p)}$ into two closed convex sets:

$$\overline{Q_{p(m-p)}^{+}}=\left\{y=(y_{j1j2})\in Q_{p(m-p)}, (j_1j_2)\in\Omega: \sum_{(j_1j_2)\in\Omega}y_1y_2>t-1\right\};$$

$$\overline{Q_{p(m-p)}^{-}}=\left\{y=(y_{j1j2})\in Q_{p(m-p)}, (j_1j_2)\in\Omega: \sum_{(j_1j_2)\in\Omega}y_1y_2\leq t-1\right\};$$

Then, there is at least t-1 1s in any one of $\overline{Q_{p(m-p)}^{-}}$, and the rest is 0; there is at most t-1 1s in any one of $\overline{Q_{p(m-p)}^{-}}$, and the rest is 0.

For each determined $(k, l)\in\Omega$,Take $\tilde{T}_{t-1}\subset\Omega\setminus\{(k,l)\}$ to meet:

$$\sum_{(j_1j_2)\in\tilde{T}_{t-1}}\|B_{kj_2}\|\|B_{lj1}\| = \max_{T_{t-1}\in\Omega\setminus\{(k,l)\}}\sum_{(j_1,j_2)\in T_{t-1}}\|B_{kj2}\|\|B_{lj1}\|$$

So for each point $y=(y_{j1j2})$ in $\overline{Q_{p(m-p)}^{-}}$ there are:

$$(\|B_{kk}^{-1}\|^{-1}-R_k^{\alpha})\left(\|A_{ll}^{-1}\|^{-1}-R_l^{(\beta)}\right) > R_{kt}^{(\beta)}R_{lt}^{(\alpha)} \geq \sum_{(j_1,j_2)\in\tilde{T}_{t-1}}\|B_{kj2}\|\|B_{lj1}\| \geq \sum_{(j_1,j_2)\in\Omega}\|B_{kj2}\|\|B_{lj1}\| y_{j1j2}$$

Then $\overline{Q_{p(m-p)}^{-}}\subseteq S_1^{+}$.

For each determined $(k,l)\in\Omega$ and $\tilde{T}_{t-1}\subset\Omega\setminus\{(k,l)\}$,let $\tilde{T}_{t-1}$ meet:

$$\sum_{(j_1,j_2)\in\tilde{T}_{t-1}}\left[\left(\|B_{j_1j_1}^{-1}\|^{-1}-C_{j_1}^{(\alpha)}\right)\left(\|B_{j_2j_2}^{-1}\|^{-1}-C_{j_2}^{(\beta)}\right)-C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}\right]$$

$$=\min_{T_{t-1}\subset\Omega\setminus\{k,l\}}\sum_{(j_1,j_2)\in\tilde{T}_{t-1}}\left[\left(\|B_{j_1j_1}^{-1}\|^{-1}-C_{j_1}^{(\alpha)}\right)\left(\|B_{j_2j_2}^{-1}\|^{-1}-C_{j_2}^{(\beta)}\right)-C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}\right].$$

For each point $y=(y_{j_1j_2})$ in A $\overline{Q_{p(m-p)}^{-}}$:

$$\sum_{(j_1,j_2)\in\tilde{T}_{t-1}\cup(k,l)}\left(\|B_{j_1j_1}^{-1}\|^{-1}-C_{j_1}^{(\alpha)}\right)\left(\|B_{j_2j_2}^{-1}\|^{-1}-C_{j_2}^{(\beta)}\right) > \sum_{(j_1,j_2)\in\tilde{T}_{t-1}\cup(k,l)}C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)},$$

Then:

$$\sum_{(j_1,j_2)\in\Omega}[\left(\left\|B_{j_1j_1}^{-1}\right\|^{-1}-C_{j_1}^{(\alpha)}\right)\left(\left\|B_{j_2j_2}^{-1}\right\|^{-1}-C_{j_2}^{(\beta)}\right)-C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}]y_{j_1j_2}\geq$$

$$\sum_{(j_1,j_2)\in\tilde{T}_{t-1}}[\left(\left\|B_{j_1j_1}^{-1}\right\|^{-1}-C_{j_1}^{(\alpha)}\right)\left(\left\|B_{j_2j_2}^{-1}\right\|^{-1}-C_{j_2}^{(\beta)}\right)-C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}]$$

$$>C_k^{(\beta)}C_l^{(\alpha)}-\left(\left\|B_{kk}^{-1}\right\|^{-1}-C_k^{(\alpha)}\right)\left(\left\|B_{ll}^{-1}\right\|^{-1}-C_l^{(\beta)}\right)$$

Thereby $\overline{Q_{p(m-p)}^{+}}\subseteq S_2^{-}$.

In summary, $\overline{Q_{p(m-p)}}=\overline{Q_{p(m-p)}^{+}}\cup\overline{Q_{p(m-p)}^{-}}\subseteq S_1^{+}\cup S_2^{-}$,

Known by Theorem 1, B is non-singular.

Theorem 4: $B=\left(B_{ij}\right)\in M_m\left(P_n\right)$, α and β are $<m>$ bipartitions, $\lambda_b\left(B\right)$ is a set of block eigenvalues of B, then:

$$\lambda_b\left(B\right)\subseteq\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4.$$

$$\Gamma_1=\bigcup_{j\in\beta}\left\{\Lambda\in P_n\left(C\right):\left\|\left(\Lambda-B_{jj}\right)^{-1}\right\|^{-1}\leq R_j^{(\beta)}\right\}\cup\bigcup_{j\in\alpha}\left\{\Lambda\in P_n\left(C\right):\left\|\left(\Lambda-B_{ii}\right)^{-1}\right\|^{-1}\leq R_i^{(\alpha)}\right\}$$

$$\Gamma_2=\left\{\Lambda\in P_n\left(C\right):\left(\left\|\left(\Lambda-B_{ii}\right)^{-1}\right\|^{-1}-R_i^{(\alpha)}\right)\left(\left\|\left(\Lambda-B_{jj}\right)^{-1}\right\|^{-1}-R_j^{(\beta)}\right)\leq R_{it}^{(\beta)}R_{jt}^{(\alpha)}\right\};$$

$$\Gamma_3=\left\{\begin{array}{l}\Lambda\in P_n\left(C\right):\sum_{(j_1j_2)\in T_i}\left(\left\|\left(\Lambda-B_{j_1j_1}\right)^{-1}\right\|^{-1}-C_{j_1}^{(\alpha)}\right)\left(\left\|\left(\Lambda-B_{j_2j_2}\right)^{-1}\right\|^{-1}-C_{j_2}^{(\beta)}\right)\\ \leq\sum_{(j_1j_2)\in T_i}C_{j_1}^{(\beta)}C_{j_2}^{(\alpha)}\end{array}\right\};$$

$$\Gamma_4=\bigcup_{i\in\langle m\rangle}\left\{\Lambda\in P_n\left(C\right):Det\left(\Lambda-B_{ii}\right)=0\right\};$$

Proof: For either $\Lambda\in\lambda_b\left(B\right)$, if $B\overline{\in}\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4$, It can be known from theorems 2 and 3. $I_m\otimes\Lambda-B$ is Non-singular. This conclusion contradicts the conclusions reached by the lemma, which explains $B\in\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4$, According to the arbitrariness of Λ: $\lambda_b\left(B\right)\in\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4$.

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References

[1] Li Y.T., Chen G. (2013). Two New Eigenvalue Inclusion Theorems of Block Matrices, Journal of Yunnan University (Natural Science Edition), 35 (3), 275-283.

[2] Xu C.L. (2015). The Inclusion Domain of Block Eigenvalues, Journal of Jilin Institute of Chemical Technology, 32 (8), 50-52.

[3] Cao H.S., Wu J.L. (2012). Estimation of Matrix Eigenvalues on Elliptic Regions, Journal of Shandong University (Science Edition), 47 (10), 49-53.

[4] Cui L.N. (2013). The Sufficient Conditions for Nonsingular H-Matrices, Journal of Jilin Institute of Chemical Technology, 30 (3), 91-93.

[5] Li Y.Y. (2014). New Territory of Eigenvalues of M-Matrices, Journal of Guizhou University (Natural Science Edition), 31 (3), 8-10.

[6] Zhang Y., Shang Y.Y., Lu Hongbin (2018). Generalized A-Diagonally Dominant Matrices Under A Class of Multi-Blocks, Journal of Beihua University (Natural Science Edition), 19 (3), 302-306.

[7] Cao Y., Jiang M.Q., Zheng Y.L. (2013). A Note on The Eigenvalue Bounds of Generalized Saddle Point Matrices for Block Triangular Preprocessing, Journal of Computational Mathematics, 35 (1), 63-73.